Progress towards Modeling Red Tides and Algal Blooms

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Abstract

Conditions in the ocean sometimes allow specific species to populate so quickly that these species form dense aggregations of individuals. Many species of microscopic algae in particular are known to form in these dense aggregations, or "blooms." Some species that form blooms are toxic to marine organisms or humans. In this paper, one method of predicting whether or not blooms will occur involves exploring the impact of grazing zooplankton on algae populations, and how the toxin produced by the phytoplankton affects those zooplankton populations. Results show an increase in toxin production yields a decrease in the graph's period and an increase in ratio between zooplankton and phytoplankton populations. Another model explores how the relationship between diffusion and patch size influences algal blooms. This paper explores these two simple models of predicting blooms and how different parameters affect the results, and which model gives more consistent outcomes. A final model involves converting the one-dimensional diffusion model to a two-dimensional model, and yields interesting changes regarding the relationship of the patch size to whether or not there will be subsequent algal bloom (or lack thereof).

1 Introduction

Algal blooms are rapid increases in the population of microscopic phytoplankton within a marine or lake [1] environment. So-called "Red Tides" are algal blooms of dinoflagellates or diatoms that produce toxins or acids that cause temporary environmental harm (affecting water quality and killing fish, birds and mammals) in the ecosystems in which they are present. Some species that cause red tides give the surrounding water a red discoloration, hence their name [2].

Grazing zooplankton have been shown to have an impact on species that cause algal blooms. Different zooplankton are affected in different ways. For some, dinoflagellates can "disrupt the mechanical and chemical sensory system" [3] of grazers. In other species of zooplankton, the grazers show no apparent side effects from the toxins, but retain high concentrations of toxins, causing a cascading effect as the toxicity increases on the trophic pathway [4] [5]. The effect of grazing on toxic dinoflagellate blooms varies from species to species, "and depends on the species composition of the grazing community" [5].

Algal blooms and red tides have been happening before recorded history, but in recent times blooms have been happening more frequently and more intensely. "Anthropogenic influences (such as nutrient run-off inducing red tide blooms and the transport of dinoflagellate cysts in ballast water of ships) have been suggested as possible causes" [6]. In lake environments, where algal blooms are nearly exclusively caused by an excess of nutrients as a result of "drainage, water run off from agricultural field" [1], models have been created to simulate how nutrients impact algal blooms [1].

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Diffusion is often used in modeling algal blooms [7] because spatial “and temporal distribution of phytoplankton is fundamentally governed by the movement of water, because they are lacking in mobility or have only weak mobility” [8].

Considering that red tides bring with them toxins that significantly affect the environment they’re in, attempting to model the threshold that dictates whether or not a bloom will occur could, if applied, help predict when different algal blooms occur and why. As the results of a red tide can affect the ecology of an environment, local fisheries and beachgoers [9], and in some cases drinking water[10], modeling blooms can help lead to counteracting the negative environmental, social, and economic effects they cause.

In this paper I explore different models with the intent of finding the best with which to predict an algal bloom. The main contribution of this study involves expanding a one-dimensional diffusion model to two dimensions, and exploring the subsequent changes to relationships between parameters that comes with adding a dimension. The rest of this paper is organized as follows: In Sec. 2 I introduce three models for this study and describe the techniques used to analyze them. In Sec. 3 I present analytic results, and in Sec. 4 I discuss my results and how I plan to move forward with research.

2 Methods

This paper practices three methods of attempting to predict algal blooms. One involves the interaction of grazing zooplankton on algae, while two others attempt to predict the occurrence of a bloom based on diffusion. For all models, computer-generated graphs and animations are plotted using Python 3.4.

2.1 Lotka-Volterra

The first, a Lotka-Volterra model pair of equations [11]:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP\left(1 - \frac{P}{K}\right) - \alpha PZ \;, \tag{1}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = \beta PZ - \mu Z - \frac{\theta P}{\gamma + P}Z \;. \tag{2}$$

This model explores the relationship between phytoplankton and grazing zooplankton. Here P and Z represent the density of phytoplankton and zooplankton population respectively. α (>0) is the specific predation rate, β (> 0) represents the ratio of biomass consumed per zooplankton for the production of new zooplankton. μ (> 0) is the mortality rate of zooplankton. θ (> 0) is the rate of toxin production per phytoplankton species and γ (> 0) is the half saturation constant [11].

The model was coded in Python. The following parameters were used: $P = 3$, $Z = 0$, $r = .175$, $\alpha = 1$, $\beta = .35$, $\mu = .0825$, $\theta = 0.2$, $K = 160$, $\gamma = .06$. The values of r, α, β, μ, and γ were chosen by averaging each of their ”Reported ranges” in Table 1 from [11]. The population ratios between P and Z were also taken from [11]. K was chosen to be 160, and the values for θ were varied in the models. Values for θ were increased by a factor of 10 for four figures, starting at 0.01 and increasing, to see the differences in results.

2.2 1-Dimensional Diffusion

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + ru \tag{3}$$

The above model [7] is a simple one dimensional diffusion model with growth.

If the function u, of x and t, is split up into two functions, each dependent upon one variable, then x and t as variables can be isolated from one another:

$$u(x,t) = a(x)b(t) \tag{4}$$

Plugging the above equation into Eq. 3 yields:

$$\frac{\partial ab}{\partial t} = D\frac{\partial^2 ab}{\partial x^2} + rab \tag{5}$$

Simplify equation so that a and b are isolated on either side and divide through by D:

$$\frac{\frac{\left(\frac{\partial b}{\partial t}\right)}{b}}{D} - \frac{r}{D} = \frac{\left(\frac{\partial a^2}{\partial^2 x}\right)}{a} \tag{6}$$

By setting each side equal to a common constant, the left and right side can be separated. Since the variables are now totally independent from one another, the partial derivatives become ordinary derivatives, and we can now solve these equations.

This process of separating variables and solving their corresponding functions makes it possible to find key relationships between parameters. An important inequality in the model arises:

$$L \geq \pi\sqrt{\frac{D}{r}} \tag{7}$$

To see in more detail the steps that lead up to this inequality, see pages 131-133 of Edward Beltrami's *Mathematical Models in the Social and Biological Sciences*.

What this inequality states is that for any given diffusivity constant and rate of algal reproduction, there is a corresponding minimum patch width at which a population of algae can sustain itself without diffusing out of the system into the surrounding water.

Using different patch widths, Eq. 3 was plotted for multiple time values using Python, and the resulting plots can be viewed in Section 3. Animations for each model were posted, accessible through a link mentioned in the description of each figure.

2.3 Two-Dimensional Diffusion

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + ru \tag{8}$$

This model is a modification to Eq. 3, and simulates the diffusion of algae across two dimensions to provide a more realistic simulation. The model in Eq. 8 was also coded using Python, and the resulting plotted images were sequenced to form a video, accessible through a link mentioned in the description of each figure. The last frame of each video is used in the figures for this paper.

3 Results

There are three main results from this study: one for each of the models employed.

Limiting cases:

In Eq's. 1) and 2, setting $Z = 0$ and $K = 160$ yields the following expected graph in Fig. 1:

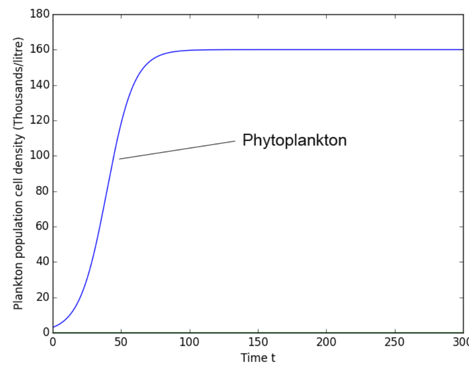

Figure 1: P (blue line) = 3, Z (green line) = 0

Similarly, setting $P = 0$ yields a similarly expected result in Fig. 2, in which the Zooplankton gradually die off since there is nothing to graze.

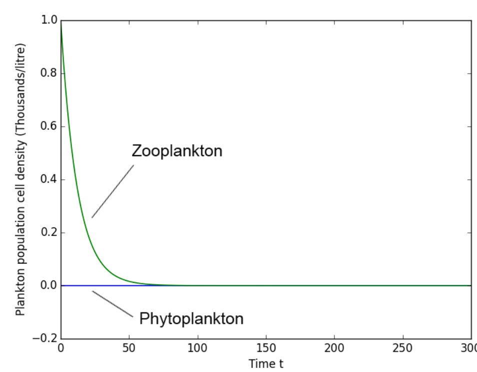

Figure 2: P (blue line) = 0, Z (green line) = 1, $\theta = 50$.

3.1 Lotka-Volterra Results

Choosing a very small value, $\theta = 0.01$, the result in Fig. 3, yields a period between oscillations of approximately 150 time units. The population of phytoplankton peaks between 4.5 and 5, and the population of zooplankton peaks at about 1.8.

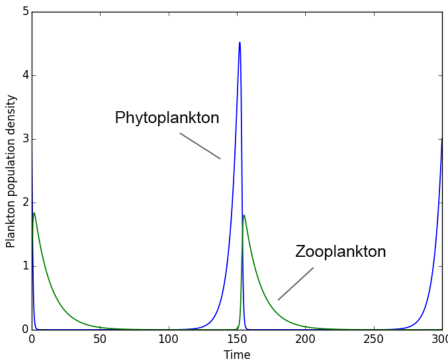

Figure 3: P (blue line) = 3, Z (green line) = 1, $\theta = 0.01$

Choosing the value $\theta = 0.1$, the result in Fig. 4 yields a period between oscillations of approximately 125 time units. The population of phytoplankton peaks slightly higher than in Fig. 3, and the population of zooplankton slightly lower.

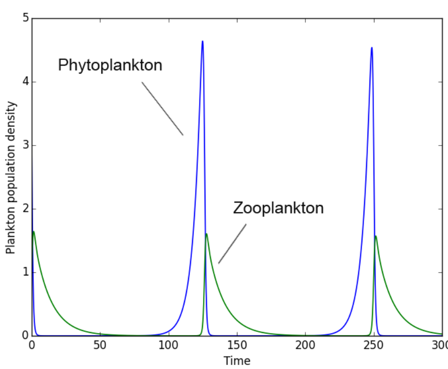

Figure 4: P (blue line) = 3, Z (green line) = 1, $\theta = 0.1$

Choosing a larger value, $\theta = 1$, the result in Fig. 5, yields a period between oscillations between 16 and 25 time units. The population of phytoplankton peaks at just above 7, and the zooplankton at about 1. The phytoplankton population dips just below 1 at its initial lowest trough.

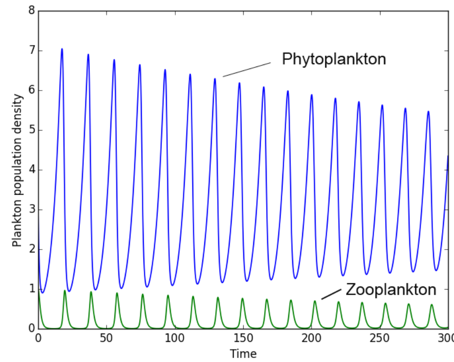

Figure 5: P (blue line) = 3, Z (green line) = 1, $\theta = 1$

Choosing a large value, $\theta = 10$, the result in Fig. 6, yields an initial spike in phytoplankton population that peaks at between 55 and 60. The phytoplankton population density does not dip below 10 at its lowest trough, and the zooplankton population density remains below one. It can be observed that equilibrium is reached at a population density approximate to 29.

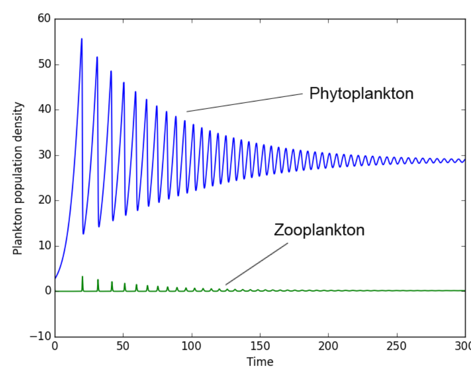

Figure 6: P (blue line) = 3, Z (green line) = 1, $\theta = 10$

It was found that as values of θ increase, the period of the graph decreases, and the ratios between the population peaks for the phytoplankton and zooplankton increases. All in all, this model is useful for observing the effects of grazing zooplankton upon red tide algae, but its results are less significant for predicting algal blooms. The impact of grazers on algal blooms is negligible when it comes to attempting to predict a bloom, and in many cases, only specific species are affected by the toxins.

3.2 1-Dimensional Diffusion Results

For the following results, these parameters were used: a starting population density of P = 50, a growth rate of r = .3, a diffusivity rate of 30, and patch width L was varied. Using these parameters, the minimum patch width to yield a bloom is calculated to be 10π.

The following occurs in Fig. 7 when the patch width is at 25, below the minimum for a bloom to occur.

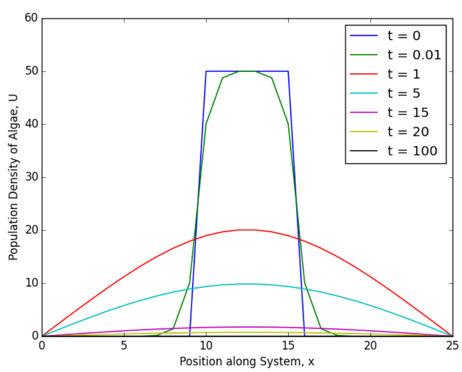

Figure 7: $L = 25$. To see the animation for for the simulation under these parameters, go to http://youtu.be/2RqTJm0IJWc

The algae quickly diffuse out of the system. There is no sign of growth - this is in accordance with the inequality.

The following occurs in Fig. 8 when the patch width is at 32, just above the minimum for a bloom to occur.

As expected, the algae do not diffuse out of the system. In fact, the algae begin to grow after the minimum population is reached, however it takes a long time for the bloom to occur.

The following occurs in Fig. 9 when the patch width is at 50, a number well above the minimum. As expected, the bloom occurs. The inequality holds true for all values tested.

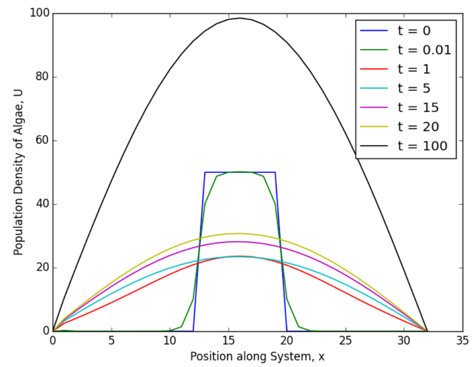

Figure 8: $L = 32$. To see the animation for this simulation, go to http://youtu.be/WKg02KqG8dA

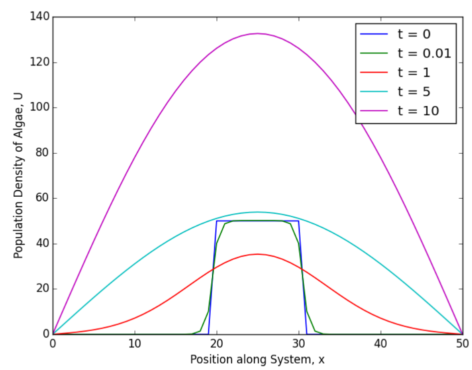

Figure 9: $L = 50$. To see the animation for this simulation, go to http://youtu.be/Kv_5Pcrp5l8

3.3 2-Dimensional Diffusion Results

The same parameters that were used in the 1-Dimensional Diffusion model were used in this one.

Fig. 10 occurs when the patch width is at 25, again, below the minimum stated by the inequality. As expected, much like in the one dimensional equivalent the algae diffuse out of the system before being able to bloom.

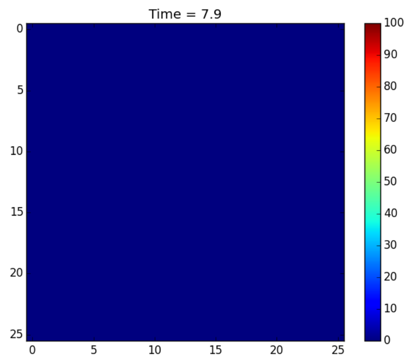

Figure 10: $L = 25$, time $= 7.9$. To see the animation for this simulation, go to `http://youtu.be/DYZxjhz1fWM`

Fig. 11 occurs when the patch width is at 32, a number just above the minimum according to the inequality.

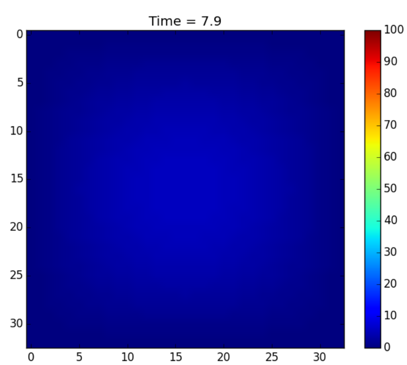

Figure 11: $L = 32$, time $= 7.9$. To see the animation for this simulation, go to `http://youtu.be/umzIcAVTb6Y`

This yields an unexpected result. After 7.9 time units, the algae have nearly completely diffused out of the system. Running the model longer shows that the algae have completely diffused.

This requires a reevaluation of the inequality. Through trial and error, I found that the minimum patch width for this two dimensional model was approximately 44.4. The ratio between the two-dimensional and one-dimensional models is about 1.413, or approximately $\sqrt{2}$.

Shown in Fig. 12 is the model with a patch width of 44.5. Much like the one dimensional model at a patch width of 32, the population density seems to stagnate at its lowest point - it grows, but very slowly, and in reality would not cause a large bloom.

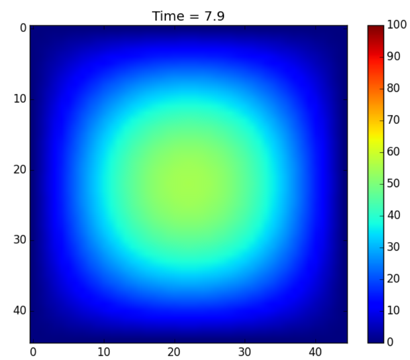

Figure 12: $L = 44.5$, time $= 7.9$. To see the animation for this simulation, go to `http://youtu.be/RMZZFXZzA2A`

To better understand the reasons for this new minimum, $\sqrt{2}$ is plugged into the inequality:

$$L\sqrt{2} \geq \pi\sqrt{\frac{D}{r}} \tag{9}$$

Rearranging the terms in order to isolate D:

$$\frac{D}{2} \leq \frac{L^2 r}{\pi^2} \tag{10}$$

Rearranging the terms in order to isolate r:

$$2r \geq \frac{D\pi^2}{L^2} \tag{11}$$

What this new inequality asserts is that for a two dimensional diffusion model, the diffusivity rate of the system must be one half of that of the one dimensional diffusion model. Alternatively, the rate of population growth must be twice that of the one dimensional diffusion model. Since in actuality the diffusivity and the rate of growth would remain relatively constant, the patch width would have to be $\sqrt{2}$ times that of the one dimensional model. These findings make sense, because the algae is now diffusing across two dimensions, and the new parameters must make up for the faster rate at which the algae is lost.

Fig. 13 occurs with a patch width of 75, well above the minimum according to the new inequality. This final run of the model verifies that the new inequality is true for numbers larger than the minimum.

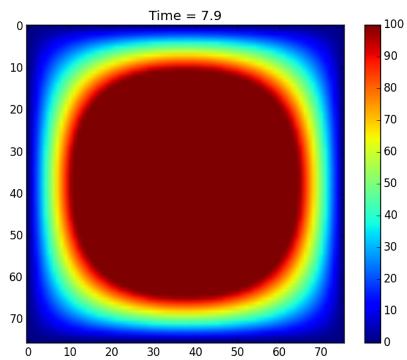

Figure 13: $L = 75$, time = 7.9. To see the animation for this simulation , go to `http://youtu.be/r9DyvT96AIY`

4 Conclusion and Future Research

The Lotka-Volterra model was helpful as far as exploring the impact of zooplankton on phytoplankton. It is not ideal for predicting the occurrence of blooms, but is useful for potentially controlling red tides through the introduction of grazing zooplankton. This model could be very useful for modeling very specific conditions of red tide; how to control specific red tide species with specific grazers.

Both the second and third models are simple, but are good ways of determining whether or not blooms will occur based on the algae population across a certain patch area. The biggest surprise in graphing these models was the unprecedented change in the inequality. It turned out to be logical, since considering the algae population is diffusing across a second dimension it must reproduce at twice the rate to sustain a bloom.

Future work will include adding more parameters to the equation, including algae death, dependence upon nutrients, adding variance in diffusivity, and potentially incorporating the Lotka-Volterra relationship into the improved diffusion model. Also among my goals are modeling diffusion across three dimensions, as well as finding the new relationship corresponding between patch size and the occurrence of blooms in three dimensions.

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